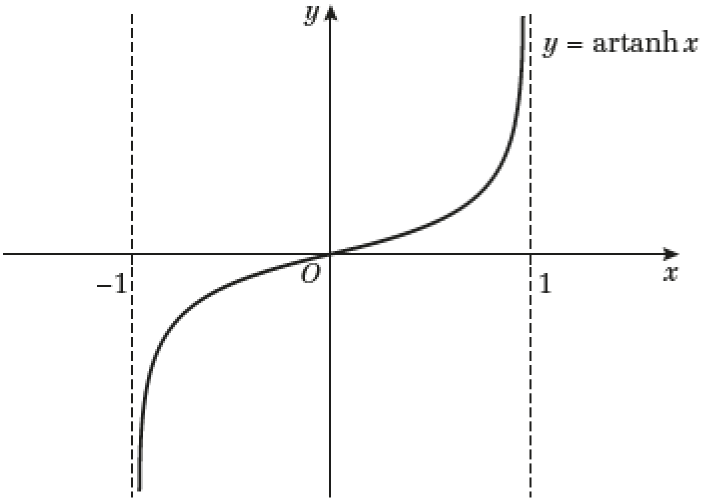
Exercise 1C

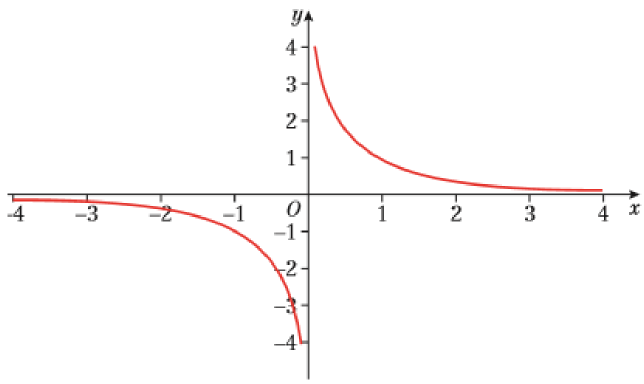
1



$y = \text{artanh}x,\ |x| < 1.$

2 $y = \text{arcosech}\,x,\ x \neq 0$

Reflect the graph of $y = \text{cosech}\,x$ in the line $y = x$



3

$y = \text{artanh}\,x$

$x = \tanh y = \dfrac{e^{2y} - 1}{e^{2y} + 1}$

$x\left(e^{2y} + 1\right) = e^{2y} - 1$

$1 + x = e^{2y}(1 - x)$

$e^{2y} = \dfrac{1 + x}{1 - x}$

$2y = \ln\left(\dfrac{1 + x}{1 - x}\right)$

$y = \dfrac{1}{2}\ln\left(\dfrac{1 + x}{1 - x}\right)$

$\text{artanh}x = \dfrac{1}{2}\ln\left(\dfrac{1 + x}{1 - x}\right),$

$|x| < 1$

For $|x| \geq 1$, $\ln\left(\dfrac{1 + x}{1 - x}\right)$ is not defined, since $\dfrac{1 + x}{1 - x} \leq 0.$

4 a $\operatorname{arsinh} 2 = \ln\left(2+\sqrt{2^2+1}\right)$

$= \ln\left(2+\sqrt{5}\right)$

b

$\operatorname{arcosh} 3 = \ln\left(3+\sqrt{3^2-1}\right)$

$= \ln\left(3+\sqrt{8}\right)$

$= \ln\left(3+2\sqrt{2}\right)$

c

$\operatorname{artanh}\left(\frac{1}{2}\right) = \frac{1}{2}\ln\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right)$

$= \frac{1}{2}\ln 3$

5 a $\operatorname{arsinh}\sqrt{2} = \ln\left(\sqrt{2}+\sqrt{2+1}\right)$

$= \ln\left(\sqrt{2}+\sqrt{3}\right)$

b $\operatorname{arcosh}\sqrt{5} = \ln\left(\sqrt{5}+\sqrt{5-1}\right)$

$= \ln\left(2+\sqrt{5}\right)$

c

$\operatorname{artanh} 0.1 = \frac{1}{2}\ln\left(\frac{1+0.1}{1-0.1}\right)$

$= \frac{1}{2}\ln\left(\frac{11}{9}\right)$

6 a $\operatorname{arsinh}(-3) = \ln\left(-3+\sqrt{(-3)^2+1}\right)$

$= \ln\left(-3+\sqrt{10}\right)$

b

$\operatorname{arcosh}\left(\frac{3}{2}\right) = \ln\left(\frac{3}{2}+\sqrt{\left(\frac{3}{2}\right)^2-1}\right)$

$= \ln\left(\frac{3}{2}+\sqrt{\frac{5}{4}}\right)$

$= \ln\left(\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$

$= \ln\left(\frac{3+\sqrt{5}}{2}\right)$

6 c

$$\text{artanh}\left(\frac{1}{\sqrt{3}}\right)=\frac{1}{2}\ln\left(\frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}\right)$$
$$=\frac{1}{2}\ln\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$$
$$=\frac{1}{2}\ln\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}\right)$$
$$=\frac{1}{2}\ln\left(\frac{4+2\sqrt{3}}{2}\right)$$
$$=\frac{1}{2}\ln\left(2+\sqrt{3}\right)$$

7

$$\text{artanh}\, x+\text{artanh}\, y$$
$$=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)+\frac{1}{2}\ln\left(\frac{1+y}{1-y}\right)$$

Use $\ln a+\ln b=\ln(ab)$.

$$=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\times\frac{1+y}{1-y}\right)$$

Use $\frac{1}{2}\ln a=\ln a^{\frac{1}{2}}$.

$$=\frac{1}{2}\ln\left(\frac{1+x+y+xy}{1-x-y+xy}\right)$$
$$=\ln\sqrt{\left(\frac{1+x+y+xy}{1-x-y+xy}\right)}$$

So $\dfrac{1+x+y+xy}{1-x-y+xy}=3$

$$1+x+y+xy=3-3x-3y+3xy$$
$$1+x-3+3x=-3y+3xy-y-xy$$
$$2xy-4y=4x-2$$
$$y(x-2)=2x-1$$
$$y=\frac{2x-1}{x-2}$$